Normal ordering solution to quantum dissipation and its induced decoherence

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We implement the normal ordering technique to study the quantum dissipation of a single mode harmonic oscillator system. The dynamic evolution of the system is investigated for a reasonable initial state by solving the Schrödinger equation directly through the normal ordering technique. The decoherence process of the system for the cases $T = 0K$ and $T \neq 0K$ is investigated as an application.

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I. INTRODUCTION

The quantum dissipation of a single mode harmonic oscillator coupled to a reservoir with an extremely large number of degrees of freedom has been intensely studied [1–5] in the past decades with various methods, such as master-equation approach, Langevin approach. By using the Markovian master-equation approach, the effect of dissipation on a macroscopic superposition of two coherent states of a harmonic oscillator was studied [6]. It shows that the superposition of the system will be transformed into a classical mixture state due to the effects of the environment [7], which is called decoherence process. The rate of the decoherence is proportional to the distance between the two states, which has been demonstrated in cavity QED experiment [8]. The wave function structure of a single mode boson system plus a bath of many bosons is studied for both factorization case [9,10] and partial factorization case [11]. It shows that when the Brownian motion effect is ignored in certain conditions the total wave function of the total system can be written in a form of a product of the bath and system components. Based on the result [11] , we have studied the decoherence process of excitons in an idealized quantum well placed in a lossy cavity [14]. The result shows that the coherence of excitonic superposition is reduced in an oscillating form due to the dissipation of the optical cavity. In references [12,13], by solving the Heisenberg equations, the state vector of a driven-damped oscillator system evolved from an initial coherent state has also solved explicitly.

As far as our knowledge is concerned, the dynamic evolution of a damped oscillator system is only investigated by implementing the Heisenberg-Schrödinger picture transformation (HSPT) [11–13]. Can we solve the Schrödinger equation directly and get the state vector of a damped harmonic oscillator system without HSPT? To answer this question, in this paper, we firstly apply normal ordering technique (NOT) to study the time evolution of a damped harmonic oscillator. The explicit form of the state vector of the total system evolved from an initially factorized coherent state is calculated. Besides, as one of the applications, we will devote ourselves to studying the decoherence behavior of the quantum dissipating system.

For zero temperature case, the decoherence time of a superposition of two coherent states due to dissipation is given in a explicit form which agrees with the previous results [6,14]. The effect of finite temperature of the reservoir on the decoherence rate is also investigated.

This paper is organized as following: in section II, we develop the normal ordering technique to solve the dynamic evolution of a damped harmonic oscillator under certain initial condition. In section III, the decoherence of a superposition state of the single mode boson system is investigated. In subsection A, the decoherence process of the system in the case of zero temperature is studied for an factorized initial state. We find that, the decoherence occurs in a time scale which is sensitive to the distance of the initial superposed states. The temperature effect on the coherence evolution of a damped oscillator is investigated in subsection B by using NOT.

II. NORMAL ORDERING TECHNIQUE

The normal ordering technique [1] is firstly introduced to study the dynamic evolution of a driven oscillator as well as that of two weakly coupled oscillators without dissipation. In this section we adopt the spirit of NOT to study the dissipation process of a single-mode boson system coupled with a reservoir.

We consider a harmonic oscillator coupled to a reservoir which involves a large collection of systems with many degrees of freedom, that is a damped oscillator. The reservoir is modeled as a harmonic oscillator bath, which in practice can be the modes of the radiation or the quantized modes of elastic vibrations (phonons) in a solid. The Hamiltonian for the system plus bath can be described as

$$H = \hbar\omega a^\dagger a + \hbar\sum_j \omega_j b_j^\dagger b_j + \hbar\sum_j (g_j a^\dagger b_j + H.c.), \tag{1}$$

where $a(a^\dagger)$ denotes the annihilation (creation) operator of the harmonic oscillator with frequency ω, and $b_j(b_j^\dagger)$ is the annihilation (creation) operator for the mode of the bath with frequency ω_j, which obey the boson commutation relations $[b_j, b_j^\dagger] = \delta_{j,j'}$. g_j denotes coupling constant between the harmonic oscillator with frequency ω and the oscillator mode of the reservoir with frequency ω_j. The model described in eq.(1) can be exactly solved as long as the coupling constant g_j and the spectrum density of the bath are specified explicitly [11]. Here we adopt the spirit of the normal ordering method to study the dynamic evolution of a single mode oscillator with dissipation.

The state vector of the whole system obeys the Schrödinger equation which has a solution of the form $|\psi(t)\rangle = U(t)\,|\psi(0)\rangle$, where the time-evolution operator $U(t)$ satisfies $i\hbar\partial_t U(t) = HU(t)$, with the initial condition $U(0) = 1$. We assume that evolution operator has its normal order form $U(t) = U^{(n)}(t)$. Since the normal form of any operator is unique, one can establish the one-to-one corresponding relationship between the normal ordered evolution operator $U^{(n)}(t)$ and an ordinary function $\overline{U}^{(n)}(t)$, with $\overline{U}^{(n)}(t) = \langle\alpha, \{\beta_j\}|\, U^{(n)}(t)\,|\{\beta_j\}, \alpha\rangle$, where $|\{\beta_j\}\rangle = |\beta_1\rangle\,|\beta_2\rangle \cdots |\beta_\infty\rangle$. Such a corresponding relation defines a map $\aleph^{-1}$,

$$\aleph^{-1} : U^{(n)}(t) \to \overline{U}^{(n)}(t) = \langle\alpha, \{\beta_j\}|\, U^{(n)}(t)\,|\{\beta_j\}, \alpha\rangle\,. \tag{2}$$

We may also define the inverse transformation $\aleph$,

$$\aleph : \overline{U}^{(n)}(t) \to U^{(n)}(t) = U(t). \tag{3}$$

One can write the Schrödinger equation in the normal ordering form. With the operator $\aleph^{-1}$, one can further take diagonal coherent state matrix elements of both sides of the normal ordered Schrödinger equation and get a c-number equation of $\overline{U}^{(n)}$,

$$\begin{aligned} i\partial_t\overline{U}^{(n)} = {} & [\omega\alpha^*\left(\alpha + \frac{\partial}{\partial\alpha^*}\right) + \sum_j \omega_j\beta_j^*\left(\beta_j + \frac{\partial}{\partial\beta_j^*}\right) + \sum_j g_j \\ & \times\alpha^*\left(\beta_j + \frac{\partial}{\partial\beta_j^*}\right) + \sum_j g_j^*\beta_j^*\left(\alpha + \frac{\partial}{\partial\alpha^*}\right)]\overline{U}^{(n)}. \end{aligned} \tag{4}$$

Now we need to solve the equation for $\overline{U}^{(n)}(t)$ with the initial condition $\overline{U}^{(n)}(0) = 1$.

We let

$$\overline{U}^{(n)} = \exp[A\alpha^*\alpha + \sum_j B_j\beta_j^*\beta_j + {\sum_{j,j'}}' B_{j,j'}\beta_j^*\beta_{j'} + \sum_j C_j\beta_j^*\alpha + \sum_j D_j\alpha^*\beta_j], \tag{5}$$

Where the prime in $\sum'$ denotes sum over index "j" and "j'" with the condition $j \neq j'$. Substituting eq.(5) into eq.(4), we get the coupled equations for the time-dependent coefficients

$$\dot{B}_j = -i\omega_j(1 + B_j), \tag{6.a}$$

$$\dot{C}_j = -i\omega_j C_j - ig_j^*(1 + A), \tag{6.b}$$

$$\dot{D}_j = -i\omega D_j - ig_j(1 + B_j) - i{\sum_{j'}}' g_{j'} B_{j',j}, \tag{6.c}$$

$$\dot{A} = -i\omega(1 + A) - i\sum_j g_j C_j, \tag{6.d}$$

$$\dot{B}_{j,j'} = -i\omega_j B_{j,j'} - ig_j^* D_{j'}, \tag{6.e}$$

with the initial condition $A(0) = B_{j,j'}(0) = C_j(0) = D_j(0) = 0$. The solution of eqs.(6) is easily obtained by using Wigner-Weisskopf approximation [1],

$$A(t) = u(t) - 1, \qquad B_j = e^{-i\omega_j t} - 1, \tag{7.a}$$

$$C_j(t) = u_j(t), \qquad D_j(t) = v_j(t), \tag{7.b}$$

$$B_{j,j'}(t) = v_{j,j'}(t), \tag{7.c}$$

where, for the bath with the frequency spectrum distribution $\rho(\omega)$

$$u(t) = e^{-i\widetilde{\omega}t} e^{-\gamma t/2}, \tag{8.a}$$

$$u_j(t) = -g_j^* e^{-i\omega_j t} \frac{e^{i(\omega_j - \widetilde{\omega})t} e^{-\gamma t/2} - 1}{\omega_j - \widetilde{\omega} + i\gamma/2}, \tag{8.b}$$

$$v_j(t) = -g_j e^{-i\omega_j t} \frac{e^{i(\omega_j - \widetilde{\omega})t} e^{-\gamma t/2} - 1}{\omega_j - \widetilde{\omega} + i\gamma/2}, \tag{8.c}$$

$$v_{j,j'}(t) = \frac{g_j^* g_{j'} e^{-i\omega_j t}}{\omega_{j'} - \widetilde{\omega} + i\gamma/2} \left(\frac{e^{i(\omega_j - \widetilde{\omega})t} e^{-\gamma t/2} - 1}{\omega_j - \widetilde{\omega} + i\gamma/2} - \frac{e^{i(\omega_j - \omega_{j'})t} - 1}{\omega_j - \omega_{j'}} \right), \tag{8.d}$$

here, $\gamma = 2\pi\rho(\omega)\left|g(\omega)\right|^2$ is the damping constant of the oscillator induced by the coupling to the environment, and $\widetilde{\omega} = \omega + \Delta\omega$ is the renormalized physical frequency , with $\Delta\omega$ is the lamb frequency shift

$$\Delta\omega = -P\int_0^\infty d\omega_j \frac{\rho(\omega_j)\left|g(\omega_j)\right|^2}{\omega_j - \omega}, \tag{9}$$

where "P" denotes the Cauthy principle part.

If, as an example, the initial state of the total system is

$$|\psi(0)\rangle = |\alpha\rangle \otimes |\{\beta_j\}\rangle\,, \tag{10}$$

where $|\{\beta_j\}\rangle = \prod_j |\beta_j\rangle$ denotes multimode coherent state of the bath. Then at any time t the total system will evolve into

$$|\psi(t)\rangle = \left|\alpha u(t) + \sum_j \beta_j v_j(t)\right\rangle \otimes \left|\left\{\beta_j e^{-i\omega_j t} + \alpha u_j(t) + {\sum_{j'}}' v_{j,j'}(t)\beta_{j'}\right\}\right\rangle, \tag{11}$$

where we have used the sum rule,

$$|\alpha u(t) + \sum_j \beta_j v_j(t)|^2 + \sum_j |\beta_j e^{-i\omega_j t} + \alpha u_j(t) + {\sum_{j'}}' v_{j,j'}(t)\beta_{j'}|^2 = |\alpha|^2 + \sum_j |\beta_j|^2, \tag{12}$$

which is derived from the normalized condition $\langle\psi(t)|\psi(t)\rangle = 1$. We see from eq.(13), due to the bath fluctuation and the back-action of system on the bath, the state vector evolved from factorized initial state becomes fully entangled. If the Brownian effect caused by the terms $\sum_j \beta_j v_j(t)$ can be ignored, the total state vector can be partially factorized [11]. We can further consider $T = 0K$ for the bath, that is, all the oscillator modes of the reservoir are in vacuum state initially. In cavity QED, this means that we neglect the background radiation of cavity. Then from eq.(13), the state vector can be simplified as

$$|\psi(t)\rangle = |\alpha u(t)\rangle \otimes |\{\alpha u_j(t)\}\rangle\,, \tag{13}$$

and the corresponding sum rule is

$$|u(t)|^2 + \sum_j |u_j(t)|^2 = 1. \tag{14}$$

III. DECOHERENCE OF SUPERPOSITION STATE OF THE HARMONIC OSCILLATOR DUE TO DISSIPATION

Recently the mesoscopic superposition state of the cavity mode states [8] has been prepared experimentally. The phase shift between two components of the superposition can be well controlled by adjusting the interaction time between atoms and the cavity mode. Then the even and odd Schrödinger cats of cavity states were prepared in the experiment, with which a new scheme for logic qubit encoding are proposed [15] to simplify the error correction circuits and improve the efficiency of the error correction in quantum computation. Decoherence process of the superposed cavity state was also demonstrated in the experiment. Due to the presence of dissipation in the cavity, the coherent information of Schrödinger cat state will be lost within a time scale which is sensitive to the size of the meter states.

As an application, in this section we will study the decoherence behavior of the damped oscillator described by eq.(1). Starting with eq.(11), we will calculate the decoherence factor, which is defined as the coefficients of the off-diagonal element of the reduced density operator of system, for both $T = 0K$ and finite temperature of the heat bath.

A. Zero-temperature case

A single mode boson system coupled to a bath which is composed of a collection of many harmonic oscillators will result in the decohence process of the system, that is, a superposition state of the system will be transformed into a statistical mixture state due to the influence of the bath. If the initial state of the total system is

$$|\Psi(0)\rangle = (C_1|\alpha_1\rangle + C_2|\alpha_2\rangle) \otimes |\{0_j\}\rangle \, , \tag{15}$$

that is, a superposition state for the system, and all the oscillator modes of the bath are in the vacuum states $|\{0_j\}\rangle$ initially. Therefore from eq.(15), the state vector of the total system at time t can be written as

$$|\Psi(t)\rangle = C_1|\alpha_1 u(t)\rangle \otimes |\{\alpha_1 u_j(t)\}\rangle + C_2|\alpha_2 u(t)\rangle \otimes |\{\alpha_2 u_j(t)\}\rangle, \tag{16}$$

where the explicit form of $u(t)$, $u_j(t)$ are given in eq.(8), and eq.(11.a), respectively. In addition, the sum rule

$$\sum_j |u_j(t)|^2 = 1 - |u(t)|^2, \tag{17}$$

has been used in eq.(16), which is derived from the condition,

$$\langle\Psi(t)|\Psi(t)\rangle = |C_1|^2 + |C_2|^2 + C_1 C_2^* \langle\alpha_2|\alpha_1\rangle + C_1^* C_2 \langle\alpha_1|\alpha_2\rangle. \tag{18}$$

The dynamic decoherence process can be investigated quantitatively by calculating the reduced density matrix $Tr_R(|\Psi(t)\rangle\langle\Psi(t)|)$ of the system at any time t. Then we will get the decoherence factor, which is defined as the coefficient of the non-diagonal elements of the reduced density matrix, such as

$$\begin{aligned} F(t) &= \prod_j \langle\alpha_1 u_j(t)|\alpha_2 u_j(t)\rangle \\ &= e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)\sum_j |u_j(t)|^2}. \end{aligned} \tag{19}$$

By using the sum rule of eq.(17), the decoherence factor becomes

$$F(t) = e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)(1 - |u(t)|^2)}. \tag{20}$$

The characteristic time τ_d of the decoherence of the superposition state is determined by the short time behavior of $|F(t)|$, that is, the norm of the decoherence factor. Within the time scale $\gamma t \ll 1$, the decoherence factor can be simplified as following,

$$F(t) = e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)\gamma t}. \tag{21}$$

If we consider that $\alpha_1 = \alpha$, and $\alpha_2 = \alpha e^{i\Delta\varphi}$, where $\Delta\varphi$ is the phase shift of the initial superposed states. Then the characteristic time is determined as following,

$$\tau_d^{-1} = 2|\alpha|^2 \gamma \sin^2(\Delta\varphi/2), \tag{22}$$

where $|\alpha|^2$ is the mean number of the oscillator. We can define the "distance" $D = |\alpha_1 - \alpha_2| = 2|\alpha|\sin(\Delta\varphi/2)$ between the two superposed states. Substituting D into eq.(22), we obtain that the characteristic time $\tau_d = \frac{2\tau_p}{D^2}$, where $\tau_p = 1/\gamma$ is the life time of the oscillator due to its energy dissipation. Our result shows that the decoherence time is determined by both the phase difference of the meter states of initial superposition state and the mean number of the quanta of the single mode boson field for a fixed damping rate. For a special case $\Delta\varphi = \pi$, which means that the system is prepared initially in an odd and even coherent states, the norm of decoherence factor of eq.(20) becomes

$$|F(t)| = e^{-2|\alpha|^2(1-e^{-\gamma t})}, \tag{23}$$

as a function of time. It shows that the coherence of the oscillator will decrease in the exponential decay rule.

B. finite temperature case

We assume that every oscillator mode of the bath is initially in thermal equilibrium state, and the single-mode boson is in a superposition of two coherent states. Then the density operator of total system at $t = 0$ reads,

$$\rho(0) = \rho_s \otimes \rho_b, \tag{24}$$

with

$$\rho_s = (C_1|\alpha_1\rangle + C_2|\alpha_2\rangle)(\langle\alpha_2|\, C_2^* + \langle\alpha_1|\, C_1^*), \tag{25a}$$

and

$$\rho_b = \int \prod_j d^2\beta_j \frac{1}{\pi\langle n_j\rangle} e^{-|\beta_j|^2/\langle n_j\rangle} \left|\beta_j\right\rangle \left\langle\beta_j\right|, \tag{25.b}$$

where $\langle n_j\rangle$ denotes the mean occupation of the j-th oscillator mode with frequency ω_j of the heat bath. From eq.(11), the decoherence factor becomes

$$F(t) = e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^* \alpha_2)(1-|u(t)|^2)} \prod_j f_j, \tag{26}$$

with

$$f_j = \int \frac{d^2\beta_j}{\pi \langle n_j \rangle} e^{-|\beta_j|^2/\langle n_j \rangle} e^{(\alpha_2^* - \alpha_1^*) u^*(t) v_j(t) \beta_j - C.c.}. \tag{27}$$

By using the identity

$$\frac{1}{\pi} \int d^2\beta \exp(-\lambda |\beta|^2 + \mu\beta + \nu\beta^*) = \frac{1}{\lambda} \exp\left(\frac{\mu\nu}{\lambda}\right), \tag{28}$$

with condition $\mathrm{Re}\{\lambda\} > 0$ and arbitrary μ and ν, we obtain f_j

$$f_j = e^{-\frac{1}{4}|\alpha_2 - \alpha_1|^2 |u(t)|^2 |v_j(t)|^2 \langle n_j \rangle}. \tag{29}$$

Substituting f_j of eq.(29) into eq.(26), we get

$$\begin{aligned} F(t) &= e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^* \alpha_2)(1-|u(t)|^2)} e^{-\frac{1}{4}|\alpha_2 - \alpha_1|^2 |u(t)|^2 \sum_j |v_j(t)|^2 \langle n_j \rangle} \\ &= e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^* \alpha_2)(1-|u(t)|^2)} e^{-\frac{1}{4}|\alpha_2 - \alpha_1|^2 |u(t)|^2 \overline{n}(1-e^{-\gamma t})}, \end{aligned} \tag{30}$$

where we have used the relation [12],

$$\sum_j |v_j(t)|^2 \langle n_j \rangle = \overline{n}(1 - e^{-\gamma t}), \tag{31}$$

here $\overline{n} = \left(e^{\frac{\hbar\omega}{k_B T}} - 1\right)^{-1}$, with k_B is the Boltzmann constant and T is the absolute temperature of heat bath. For low temperature, $\overline{n} \sim 0$, the decoherence factor of eq.(30) will go back eq.(20). While at high temperature, $\overline{n} \sim \frac{k_B T}{\hbar\omega}$, as the previous subsection, the characteristic time τ_d of the decoherence of the superposition state is determined by calculating the norm of the decoherence factor within the time scale $\gamma t \ll 1$, that is,

$$F(t) = e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^* \alpha_2)\gamma t} e^{-\frac{1}{4}\frac{k_B T}{\hbar\omega}|\alpha_2 - \alpha_1|^2 \gamma t}. \tag{32}$$

For $\alpha_1 = \alpha$, and $\alpha_2 = \alpha e^{i\Delta\varphi}$, with $\Delta\varphi$ is the phase shift between the "meter" states of the initial superposition state. Then the decoherence time is determined as

$$\tau_d^{-1} = 2|\alpha|^2 \gamma \left(1 + \frac{k_B T}{2\hbar\omega}\right) \sin^2(\Delta\varphi/2). \tag{33}$$

As the previous subsection we substitute the "distance" $D = |\alpha_1 - \alpha_2| = 2|\alpha| \sin(\Delta\varphi/2)$ between the two superposed states into eq.(33) and obtain the decoherence time $\tau_d = \frac{2\tau_p}{D^2}\frac{1}{1+\frac{k_B T}{\hbar\omega}}$. Comparing with the zero temperature case, we find that due to the finite temperature effect of bath the decoherence rate of the system becomes faster.

IV. CONCLUSION

The quantum dissipation process of a single-mode boson immersed in a bath of bosons is studied by using the normal ordering technique. The dynamic evolution of the total system is obtained and is used to study the decoherence behavior of the system both for the zero temperature case and for the finite temperature case. Due to the influence of the dissipation the coherence information of the system will be lost in a time scale which is dependent on the distance of the initial superposition state. For $T \neq 0K$, the finite temperature effect of the bath will decrease the decoherence time. One can note that the normal ordering method can be also used to study the dynamic evolution of a driven-damped oscillator system.

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